11.4: 4, 14* (graph not necessary!), 26**(use linear approx, not differentials), 36, 38

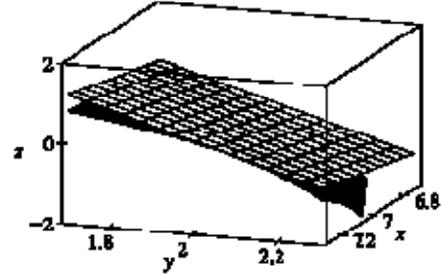**4.** $z = f(x,y) = y \ln x \Rightarrow f_x(x,y) = y/x$, $f_y(x,y) = \ln x$, so $f_x(1,4) = 4$, $f_y(1,4) = 0$, and an equation of the tangent plane is $z - 0 = f_x(1,4)(x - 1) + f_y(1,4)(y - 4) \Rightarrow z = 4(x - 1) + 0(y - 4)$ or $z = 4x - 4$.

14. $f(x,y) = \ln(x - 3y) \Rightarrow f_x(x,y) = \dfrac{1}{x - 3y}$ and $f_y(x,y) = -\dfrac{3}{x - 3y}$, so $f_x(7,2) = 1$ and $f_y(7,2) = -3$.

Then the linear approximation of f at $(7,2)$ is given by

$$f(x,y) \approx f(7,2) + f_x(7,2)(x - 7) + f_y(7,2)(y - 2)$$

$$= 0 + 1(x - 7) - 3(y - 2) = x - 3y - 1$$

Thus $f(6.9, 2.06) \approx 6.9 - 3(2.06) - 1 = -0.28$. The graph shows that our approximated value is slightly greater than the actual value.
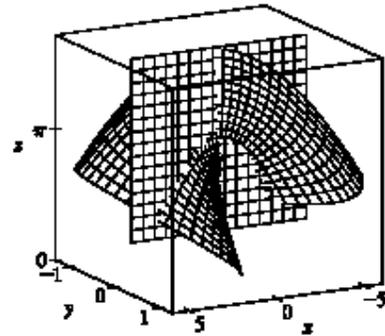

26. Let S be surface area. Then $S = 2(xy + xz + yz)$ and $dS = 2(y + z)\,dx + 2(x + z)\,dy + 2(x + y)\,dz$. The maximum error occurs with $\Delta x = \Delta y = \Delta z = 0.2$. Using $dx = \Delta x$, $dy = \Delta y$, $dz = \Delta z$ we find the maximum error in calculated surface area to be about $dS = (220)(0.2) + (260)(0.2) + (280)(0.2) = 152\text{ cm}^2$.

36. $\mathbf{r}(u,v) = uv\,\mathbf{i} + u \sin v\,\mathbf{j} + v \cos u\,\mathbf{k} \Rightarrow \mathbf{r}(0,\pi) = (0,0,\pi)$.

$\mathbf{r}_u = v\,\mathbf{i} + \sin v\,\mathbf{j} - v \sin u\,\mathbf{k}$ and $\mathbf{r}_v = u\,\mathbf{i} + u \cos v\,\mathbf{j} + \cos u\,\mathbf{k}$, so a normal vector to the surface at the point $(0,0,\pi)$ is

$\mathbf{r}_u(0,\pi) \times \mathbf{r}_v(0,\pi) = (\pi\,\mathbf{i}) \times (\mathbf{k}) = -\pi\,\mathbf{j}$. Thus an equation of the tangent plane is $-\pi(y - 0) = 0$ or $y = 0$.

38. $\mathbf{r}_1(t) = \langle 2 + 3t, 1 - t^2, 3 - 4t + t^2 \rangle \Rightarrow \mathbf{r}_1'(t) = \langle 3, -2t, -4 + 2t \rangle$,

$\mathbf{r}_2(u) = \langle 1 + u^2, 2u^3 - 1, 2u + 1 \rangle \Rightarrow \mathbf{r}_2'(u) = \langle 2u, 6u^2, 2 \rangle$. Both curves pass through P since

$\mathbf{r}_1(0) = \mathbf{r}_2(1) = \langle 2, 1, 3 \rangle$, so the tangent vectors $\mathbf{r}_1'(0) = \langle 3, 0, -4 \rangle$ and $\mathbf{r}_2'(1) = \langle 2, 6, 2 \rangle$ are both parallel to the tangent plane to S at P. A normal vector for the tangent plane is $\mathbf{r}_1'(0) \times \mathbf{r}_2'(1) = \langle 3, 0, -4 \rangle \times \langle 2, 6, 2 \rangle = \langle 24, -14, 18 \rangle$, so an equation of the tangent plane is $24(x - 2) - 14(y - 1) + 18(z - 3) = 0$ or $12x - 7y + 9z = 44$.